PART I:

1. A block weighing 15.0 N, which can slide without friction on an incline at an angle $\theta = 45.0°$, is connected to the top of the incline by a massless spring of unstretched length 0.50 m and spring constant 94.5 N/m. If the block is pulled slightly down the incline and released, the period of the resulting oscillations is:

   $W = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{W} = 2\pi\sqrt{\frac{m}{K}} = 0.79$

   a. 0.8 s
   b. 0.6 s
   c. 0.7 s
   d. 1.8 s
   e. None of the above, my answer is ____________

2. A simple, harmonic oscillator at the point $x = 0$ generates a wave on a rope. The oscillator operates at a frequency of 40.0Hz and with an amplitude of 3.00cm. The mass of the rope per unit length is 50.0 g/m and the tension in the rope is 5.00 N. The maximal transverse acceleration of the point on the rope is:

   $a_m = y_m W^2 = y_m (2\pi f)^2 = 0.03(2\pi \times 40)^2$

   a. $-1.89\ m/s^2$
   b. $1.89\ m/s^2$
   c. $189\ m/s^2$
   d. $18.9\ m/s^2$
   e. None of the above, my answer is 1894.9 m/s^2

3. A string along which waves can travel is 2.70 m long and has a mass of 270 g. The tension in the string is 3.60 N. The frequency of traveling waves of amplitude 7.00 mm is 200 Hz. The corresponding average kinetic energy for the later frequency is:

   $K = \frac{1}{4} y_m^2 W^2 \mu \lambda = \frac{1}{4}(0.007)^2(2\pi \times 200)^2\left(\frac{0.27}{2.7}\right)\left(\frac{1}{f}\sqrt{\frac{3.6}{\frac{0.27}{2.7}}}\right)$ $\rightarrow \lambda$

   a. 23.213 mJ
   b. 2352 mJ
   c. 11.607 mJ
   d. 1176 mJ
   e. None of the above, my answer is 0.058 J

4.

4. The amplitudes of two sinusoidal string waves are 2.0 cm and 3.0 cm, and they have the phase constants of $\frac{\pi}{6}$ and $\frac{2\pi}{3}$ rad, respectively. They have the same frequency and they are traveling in the same direction and medium. When they are combined, the amplitude of the resultant wave is:

a. 3.6 cm
b. 5.0 cm
c. 13 cm
d. 10 cm
e. None of the above, my answer is ____________

$y^2 = y_1^2 + y_2^2 + 2 y_1 y_2 \cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right)$
$= 0.02^2 + 0.03^2 + 2(0.02)(0.03)\cos\left(\frac{\pi}{2}\right)$
$\Rightarrow y = 0.036$ m

6 pts

5. A platinum wire has a linear density of 8.5×10^{-4} kg/m and a length $L = 0.450$ m. One end of the wire is attached to the ceiling, while a 420g mass is attached to the other end such that the wire hangs vertically under tension. If a vibrating tuning fork of just the right frequency is held next to the wire, the wire begins to vibrate as well (in resonance). What must be the frequency of the tuning for that to happen?

a. 154.64 Hz
b. 77.32 Hz
c. answers a and b
d. None of the above, my answer is ____________

$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$
$= \frac{n}{2 \times 0.45}\sqrt{\frac{0.42 \times 9.81}{8.5 \times 10^{-4}}} = n\,77.\ldots$

6 pts

6. A man needs to know the height of a tower, but darkness obscures the ceiling. He knows however, that a long pendulum extends from the ceiling almost to the floor and that the period is 12.0 seconds. How tall is the tower?

a. 3.5 m
b. 3.7 m
c. 35.8 m
d. None of the above, my answer is ____________

$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = 4\pi^2 \frac{L}{g}$
$\Rightarrow L = \frac{gT^2}{4\pi^2} = 35.8$

6 pts

7. Two identical waves travel in opposite directions along a string with a speed of 10 cm/s. If the time interval between instants when the string is flat is 0.5 s, the wavelength of the waves is:

a. 10 cm
b. 5.0 cm
c. 10.0 cm
d. 20.0 cm
e. None of the above, my answer is ____________

$v = \frac{\lambda}{T} \Rightarrow \lambda = vT$
$T = 2 \times 0.5$
$\Rightarrow \lambda = 10 \times 1$

6 pts

8. Two identical waves, moving in the same direction along a stretched string, interfere with each other. The combined wave has an amplitude 1.5 times that of the common amplitude of the two combining waves. The phase difference between the two waves is:

   a. 0.732 rad
   b. -0.732 rad
   c. 41.4°
   d. $\pm 0.23\lambda$
   e. None of the above, my answer is 82.8°

6 pts

9. The balance wheel of a watch oscillates with an angular amplitude of π rad and a period of 1.00 s. The angular speed at a displacement $\frac{\pi}{6}$ rad is:

   a. ± 19.463 rad/s
   b. 3.290 rad/s
   c. 19.463 rad/s
   d. None of the above, my answer is ____________

6 pts

10. A particle has a displacement $x = 0.5\cos\left(3t + \frac{\pi}{4}\right)$, where x is in meters and t is in seconds. At what time does the maximum velocity first occur?

   a. 1.31 s
   b. 0 s
   c. 131 s
   d. 0.26 s
   e. None of above, my answer is ______________

6 pts

PART II

Problem 1

Adjacent antinodes of a standing wave on a string with fixed ends are 15.0 cm apart. A particle on the string (or a segment of the string) at an antinode oscillates in a simple harmonic motion with an amplitude of 0.85 cm and period of 0.0750s.

- **a.** Find the wavelength and the period.
- **b.** Find the displacement of an arbitrary point on the string as a function of position and time.
- **c.** Find the positions of the first, second and third antinodes, and the positions of the first, second and third nodes.
- **d.** Find the displacements of segments at points 3.0 cm to the right of the first antinode and to the right of the second anti-node.

a) $\lambda = 2 \times 0.15 = 0.3\ m$ 2½ pts

$K = \frac{2\pi}{\lambda} = 21$; $W = \frac{2\pi}{T} = 83.7$

b) ~~$y = y_m \sin(Kx - wt)$~~

$y = 2 y_m \sin(Kx)\cos(wt)$

$y = 2 \times 0.0085 \sin(21x)\cos(83.7t)$ 5 pts

c) antinode ⇒ $\sin(21x) = 1 \Rightarrow 21x = (n + \frac{1}{2})\pi \Rightarrow x = (n + \frac{1}{2})\frac{\lambda}{2}$, $n(1, 2 \ldots)$

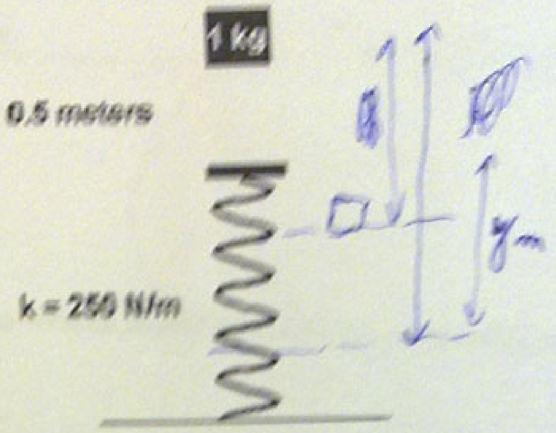
* antinode 1st: $0.075\ m = x$
  2nd: $x = 0.225\ m$
  3rd: $x = 0.375\ m$

* node: $y = 0 \Rightarrow \sin(Kx) = 0 \Rightarrow Kx = n\pi \Rightarrow x = \frac{n\pi}{2\pi}\lambda = n\frac{\lambda}{2}$
  1st: $x = 0\ m$
  2nd: $x = 0.15\ m$
  3rd: $x = 0.30\ m$

4 pts

Problem 2

Suppose a 1-kg block is dropped from a height of 0.5 meter above an uncompressed spring. The spring has an elastic constant of 250 N/m and negligible mass. The block strikes the end of the spring and sticks to it. (Use g = 10 m/sec^2)



a. How far will the spring be compressed when the speed of the block is maximum? Deduce the equilibrium position as measured from the initial position of the spring as shown in the figure.

b. Determine the spring's maximum compression x (This will occur when all of the block's energy has been converted into elastic potential energy, or use energy conservation).

c. As the spring oscillates in SHM, what is its amplitude?

d. What is the frequency of this spring/block system?

a) $\Sigma F = 0$

$Ky = mg \Rightarrow y = \frac{mg}{K} = 0.0394$ m under the initial position (5 pts)

b) $\frac{1}{2} K y_m^2 = mg(y_m + 0.5)$

~~$mg(0.5 + 0.0394)$~~

$mg(y_m + 0.5) = \frac{1}{2} K y_m^2 \Rightarrow mg y_m + mg\,0.5 = \frac{1}{2} K y_m^2$ (4 pts)

$\Rightarrow \frac{1}{2} K y_m^2 - mg y_m - mg\,0.5 \Rightarrow y_m = \text{---}$ m

c) ~~$2\pi\sqrt{\frac{m}{K}}\ y_m$~~ The amplitude is $y_m - 0.0394$ (6)

d) $f = \frac{w}{2\pi} = \sqrt{\frac{K}{m}}/2\pi = 2.516$ Hz (5 pts)